Name:______________________________

Math 3053 Section 01 | *Practice Exam 2* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

The items you must absolutely know for the exam:

(a) Truth tables, in particular how the conjunction, disjunction, negation, implication, and biconditional work with the truth values

(b) How to prove things using contradiction and direct methods

(c) How to prove things using induction

(d) The definition of a limit of a function at a point

(e) How to prove limits using the definition

1. Let P and Q be statements. Prove that the following statements are always true:

(a) $[P \wedge (P \Rightarrow Q)] \Rightarrow Q$

(b) $[P \Rightarrow (Q \wedge \neg Q)] \Rightarrow \neg P$.

2. Let P and Q be statements. Prove that P and Q are propositionally equivalent if and only if $P \Leftrightarrow Q$.

3. Prove that $\sqrt{10}$ is irrational.

4. Prove there exists irrational numbers x and y such that x^y is rational.

5. Prove that for any $n \in \mathbb{N}$ the following holds:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

6. Let $n \in \mathbb{N}$. Prove that

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

7. Suppose f and g are functions defined on an open interval I such that $a \in I$. Let

$$\lim_{x \to a} f(x) = L \text{ and } \lim_{x \to a} g(x) = M$$

Prove from the definition of the limit that

$$\lim_{x \to a} f(x)g(x) = LM$$

8. Suppose f and g are functions defined on an open interval I such that $a \in I$. Let

$$\lim_{x \to a} f(x) = L \text{ and } \lim_{x \to a} g(x) = M$$

Prove from the definition of the limit that

$$\lim_{x \to a} (f(x) + g(x)) = L + M$$

9. Define $H(x)$ as follows:

$$H(x) = \begin{cases} 5 & \text{for } x < 0 \\ 0 & \text{for } x \geq 0 \end{cases}$$

Prove that $H(x)$ does not have a limit at $x = 0$.